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STUDY OF THE EXISTENCE OF THE GLOBAL SOLUTION TO THE SYSTEM OF EQUATIONS OF THE VERTICAL MOTION OF THE AIR IN A CHIMNEY

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Abstract. In this paper, we consider the system of equations that describes air motion in a chimney. By introducing a method based on the «second approximation», we prove a theorem on the existence and uniqueness of the global solution.
Keywords: global solution; motion of the air

Introduction

In [1] we have constructed a numerical solution to the system of equations describing the chimney's air motion caused by the latent heat of condensing water vapor. The numerical construction of the solution has been made by using the approximation of separating the temporal evolution and the vertical structure. However the question of the existence and uniqueness of the solution of this system of equations, including that of the sub-system for the vertical structure, has not been solved yet.

In [2] the author proposes a theorem of the existence and uniqueness of a system of equations similar to the sub-system for the spatial structure studied in [1]. The result in [2] advances the classical theorem of the existence and uniqueness of the local solution (For an example see [3]). However, this theorem only provides the existence and uniqueness of the solution in an interval, which seems too small to guarantee the solubility of a system of equations of the kind proposed in [1].

In the present work, not only did we use the idea of [2], but we also introduced another method based on the " second approximation". We prove a theorem of the existence and uniqueness of the solution of a nonlinear system for ordinary differential equations, a theorem which develops the result of [2].

The system of ordinary differential equations that we will consider corresponds to the sub-system of vertical structure studied in [3]. We find this type of equations always in the study of the motion of the air caused by the latent heat of condensation of the water vapor as storms. This has captured most researchers' attention and maintained their interest (see [4–7]).

1. Main concepts

We denote by T the temperature of the air, ϱ the density of the air, $v = \alpha w$ the velocity of the air, g the gravitational acceleration, R_1 the constant of gas divided by the molar mass, c_v specific heat , $\overline{\pi}_{vs}(T)$ the density of saturated vapor in the air, Σ is the quantity of liquid or solid water in the air. We propose the following system of equations (see [1])

$$w\frac{d\varrho}{dz} + \varrho\frac{dw}{dz} = -H_{tr} \tag{1.1}$$

$$\varrho c_v\frac{dT}{dz} - R_1 T\frac{d\varrho}{dz} = L_{tr}h_{tr}, \tag{1.2}$$

$$\alpha(t)^2\varrho w\frac{dw}{dz} + R_1\varrho\frac{dT}{dz} + R_1 T\frac{d\varrho}{dz} = -g\varrho - g\Sigma, \tag{1.3}$$

in the domain $0 < z < \overline{z}_1$, $t \geq 0$, to model the motion of the air in a vertical cylinder of height $\overline{z}_1$, such that $\alpha(t)$ is a function of $t \geq 0$, while $\varrho = \varrho(t;z)$, $T = T(t;z)$, $w = w(t;z)$ are functions of $0 < z < \overline{z}_1$ which depend on the parameter $t \geq 0$, L_{tr} is the latent heat due to the phase transition of water from the gas state to the liquid or solid state , H_{tr} is the quantity of condensation defined by the relation

$$H_{tr} = \Big(\overline{\pi}_{vs}(T)\frac{d}{dz}\log\varrho - \frac{d}{dz}\overline{\pi}_{vs}(T)\Big)w. \tag{1.4}$$

For the pressure p , we assumed that it is given by the law valid for a perfect gas

$$p = R_1 T\varrho.$$

The system of equations (1.1)–(1.3) must be considered with the boundary conditions

$$\varrho|_{z=0} = \varrho_0, \quad T|_{z=0} = T_0, \quad w|_{z=0} = 1. \tag{1.5}$$

In order that the problem is significant from the physical point of view, the values of ϱ_0 , T_0 , w_0 should not be arbitrary, but must correspond to the physical reality of the atmosphere, so that the solution represent a real evolution of the upward flow of the air: ϱ_0 and T_0 must be in the neighborhood of values

$$\varrho_0 \approx 1204(g/m^3), \qquad T_0 \approx 300(^oK), \qquad w_0 \approx 1.$$

2. Semi-stationary problem – First approximation

We denote by φ the probability of permanence of droplets in the air, in the system of equations (1.1)–(1.3), we suppose $\alpha(t)$ and
$\Sigma = \frac{1}{\overline{z}_1}\int_0^t \varphi(t-s)\int_0^{\overline{z}_1}\Big(\overline{\pi}_{vs}(T)\frac{d}{dz}\log\varrho - \frac{d}{dz}\overline{\pi}_{vs}(T)\Big)v\,dz\,ds$ are given, so that the equations (1.1)–(1.3) become stationary (in the previous writing t must be considred as a parameter).

Let us now introduce the first approximation. We suppose the following relations

$$\varrho_{hs}^* c_v \frac{dT_{hs}^*}{dz} - R_1 T_{hs}^* \frac{d\varrho_{hs}^*}{dz} = \Big(R_1 T_{hs}^* + L_{tr}\Big)\Big(\overline{\pi}_{vs}(T_{hs}^*)\frac{d}{dz}\log \varrho_{hs}^* - \frac{d}{dz}\overline{\pi}_{vs}(T_{hs}^*)\Big), \tag{2.1}$$

$$R_1 \frac{d}{dz}(\varrho_{hs}^* T_{hs}^*) = -g\varrho_{hs}^*, \tag{2.2}$$

where $(\varrho_{hs}^*(z), T_{hs}^*(z))$ are the hydrostatic distributions of the density and the temperature of humid air. Functions $(\varrho_{hs}^*(z), T_{hs}^*(z))$, with the initial conditions

$$\varrho_{hs}^*(0) = \varrho_0, \qquad T_{hs}^*(0) = T_0,$$

is such that relations (2.1), (2.2) hold. The couple of functions $(\varrho_{hs}^*(z), T_{hs}^*(z))$ will be used as a first approximation of our solution.

3. Second approximation

Now we propose to construct the second approximation.

We suppose

$$h_{tr}^*(z) = \overline{\pi}_{vs}(T_{hs}^*(z))\frac{d}{dz}\log \varrho_{hs}^*(z) - \frac{d}{dz}\overline{\pi}_{vs}(T_{hs}^*(z)). \tag{3.1}$$

Let us define the following relations

$$\frac{d}{dz}\log \tilde{U}(z) = -\frac{h_{tr}^*}{\varrho_{hs}^*} \equiv \widetilde{f}_1, \tag{3.2}$$

$$\frac{d}{dz}\tilde{V}(z) = -\alpha(t)^2 \widetilde{U}\frac{d}{dz}\frac{\widetilde{U}}{\varrho_{hs}^*} - g\Sigma \equiv \widetilde{g}_1, \tag{3.3}$$

$$\frac{d}{dz}\tilde{W}(z) = \frac{(R_1 T_{hs}^* + L_{tr})}{\varrho_{hs}^* T_{hs}^*}\overline{\pi}_{vs}({T_{hs}}^*)\frac{d}{dz}\log {\varrho_{hs}}^* - \frac{R_1}{\varrho}\frac{d}{dz}\overline{\pi}_{vs}({T_{hs}}^*) + \frac{d}{dz}\Big(\frac{L_{tr}}{\varrho_{hs}^* T_{hs}^*}\Big)\overline{\pi}_{vs}({T_{hs}}^*) \equiv \widetilde{h}_1, \tag{3.4}$$

where the functions $(\widetilde{U}, \widetilde{V}, \widetilde{W})$ with conditions

$$\widetilde{U}(0) = \varrho_0, \quad \widetilde{V}(0) = R_1 T_0 \varrho_0, \quad \widetilde{W}(0) = \log \frac{T_0^{c_v}}{\varrho_0^{R_1}} + \frac{L_{tr}}{\varrho_{hs}^*(0) T_{hs}^*(0)}\overline{\pi}_{vs}({T_{hs}}^*(0)), \tag{3.5}$$

are such as relations (3.2)–(3.4) hold.

4. Main results

Lemma 4.1. *Let* $\widetilde{U}$, $\widetilde{V}$ *and* $\widetilde{W}$ *be the functions which satisfy equations* (3.2)–(3.4) *and conditions* (3.5).

Then there exist functions $(\widetilde{\varrho}, \widetilde{T}, \widetilde{w})$ *satisfying the relations*

$$\widetilde{U}(z) = \widetilde{\varrho}(z)\widetilde{w}(z), \;\; \widetilde{V}(z) = R_1 \widetilde{\varrho}(z)\widetilde{T}(z) + g\int_0^z \widetilde{\varrho}(z')dz', \;\; \widetilde{W}(z) = \log \frac{\widetilde{T}(z)^{c_v}}{\widetilde{\varrho}(z)^{R_1}} + \frac{L_{tr}}{\varrho_{hs}^* T_{hs}^*}\overline{\pi}_{vs}({T_{hs}}^*). \tag{4.1}$$

And they are unique.

P r o o f. It follows from the third equation of (4.1) that

$$\widetilde{\varrho}(z) = e^{-\widetilde{W}(z)/R_1}\left(\widetilde{T}(z)e^{\left(\frac{L_{tr}\overline{\pi}vs(\widetilde{T})}{(R_1+c_v)\widetilde{\varrho}\widetilde{T}}\right)}\right)^{c_v/R_1} e^{\left(\frac{L_{tr}\overline{\pi}vs(\widetilde{T})}{(R_1+c_v)\widetilde{\varrho}\widetilde{T}}\right)},$$

substituting this equality in the second equation of (4.1), we have

$$\widetilde{V}(z)=R_1e^{-\widetilde{W}(z)/R_1}\widetilde{T}(z)^{\frac{R_1+c_v}{R_1}}e^{\left(\frac{L_{tr}\overline{\pi}vs(\widetilde{T})}{R_1\widetilde{\varrho}\widetilde{T}}\right)}+g\int_0^z e^{-\widetilde{W}(z)/R_1}\left(\widetilde{T}(z)e^{\left(\frac{L_{tr}\overline{\pi}vs(\widetilde{T})}{(R_1+c_v)\widetilde{\varrho}\widetilde{T}}\right)}\right)^{c_v/R_1} e^{\left(\frac{L_{tr}\overline{\pi}vs(\widetilde{T})}{(R_1+c_v)\widetilde{\varrho}\widetilde{T}}\right)}dz'.$$

By posing $\widetilde{y}(z) = e^{-\widetilde{W}(z)/R_1}\widetilde{T}(z)^{\frac{R_1+c_v}{R_1}}e^{\left(\frac{L_{tr}\overline{\pi}vs(\widetilde{T})}{R_1\widetilde{\varrho}\widetilde{T}}\right)}$, we obtain

$$\frac{d}{dz}\widetilde{y}(z) = -\frac{g}{R_1}e^{-\widetilde{W}(z)/(R_1+c_v)}e^{\left(\frac{L_{tr}\overline{\pi}vs(\widetilde{T})}{(R_1+c_v)\widetilde{\varrho}\widetilde{T}}\right)}\widetilde{y}(z)^{\frac{c_v}{R_1+c_v}} + \frac{\widetilde{g}_1(z)}{R_1}, \qquad \widetilde{y}(0) = \varrho_0 T_0,$$

or

$$\frac{d}{dz}\widetilde{y}(z)^{\frac{R_1}{R_1+c_v}} = -\frac{g}{R_1}e^{-\widetilde{W}(z)/(R_1+c_v)}e^{\left(\frac{L_{tr}\overline{\pi}vs(\widetilde{T})}{(R_1+c_v)\widetilde{\varrho}\widetilde{T}}\right)} + \frac{\widetilde{g}_1(z)}{R_1\widetilde{y}(z)^{\frac{c_v}{R_1+c_v}}}, \qquad \widetilde{y}(0) = \varrho_0 T_0. \tag{4.2}$$

We denote

$$\widetilde{Y} = \widetilde{y}^{\frac{R_1}{R_1+c_v}}, \quad \phi(\widetilde{Y}) = e^{\left(\frac{L_{tr}\overline{\pi}vs(\widetilde{T})}{(R_1+c_v)\widetilde{\varrho}\widetilde{T}}\right)},$$

according to the Taylor formula we have

$$\frac{d\phi}{d\widetilde{Y}} = \phi(Y_0) + \frac{d\phi}{d\widetilde{Y}}\mid_{\widetilde{Y}=Y_0}(\widetilde{Y}-Y_0) + R(\widetilde{Y}).$$

We substitute in the equation (4.2) and obtain

$$\frac{d}{dz}\widetilde{Y} + \frac{g}{R_1}e^{-\frac{\widetilde{W}}{(R_1+c_v)}}\frac{d\phi}{d\widetilde{Y}}\mid_{\widetilde{Y}=Y_0}\widetilde{Y} = \frac{1}{R_1}\widetilde{g}_1\widetilde{y}^{\frac{-c_v}{R_1+c_v}} + \frac{g}{R_1}e^{-\frac{\widetilde{W}}{(R_1+c_v)}}\left(\phi(Y_0) + Y_0\frac{d\phi}{d\widetilde{Y}}\mid_{\widetilde{Y}=Y_0} + R(\widetilde{Y})\right),$$

$$\widetilde{Y}(0) = Y_0.$$

The solution of this Cauchy problem is defined by

$$\widetilde{Y}(z) = Y_0 e^{\int_0^z \frac{g}{R_1}e^{-\frac{\widetilde{W}}{(R_1+c_v)}}\frac{d\phi}{d\widetilde{Y}}\mid_{\widetilde{Y}=Y_0}dz'} +$$

$$+\int_0^z\left(\frac{1}{R_1}\widetilde{g}_1\widetilde{y}^{\frac{-c_v}{R_1+c_v}} + \frac{g}{R_1}e^{-\frac{\widetilde{W}}{(R_1+c_v)}}\left(\phi(Y_0)+Y_0\frac{d\phi}{d\widetilde{Y}}\mid_{\widetilde{Y}=Y_0}+R(\widetilde{Y})\right)\right)e^{\int_0^{z'}\frac{g}{R_1}e^{-\frac{\widetilde{W}}{(R_1+c_v)}}\frac{d\phi}{d\widetilde{Y}}\mid_{\widetilde{Y}=Y_0}dz''}dz'.$$

Found $\widetilde{Y}(z)$ the solution of the Cauchy problem (4.2), we can define immediately $\widetilde{T}$. Being constructed $\widetilde{T}$, we can define $\widetilde{\varrho}(\widetilde{T})$ as following

$$\widetilde{\varrho} = e^{-\widetilde{W}(z)/R_1}\left(\widetilde{T}(z)e^{\left(\frac{L_{tr}\overline{\pi}vs(\widetilde{T})}{(R_1+c_v)\widetilde{\varrho}\widetilde{T}}\right)}\right)^{c_v/R_1} e^{\left(\frac{L_{tr}\overline{\pi}vs(\widetilde{T})}{(R_1+c_v)\widetilde{\varrho}\widetilde{T}}\right)}$$

and $\widetilde{w}(z)$ by the relation (4.1).

The functions $(\widetilde{\varrho}, \widetilde{T}, \widetilde{w})$ thus constructed will be used as the Second approximation.

5. Priori estimates

Let us define the family of distances

$$\kappa(\varphi_1,\varphi_2)(z)=\max\big(\sup_{0\le z'\le z}|\varphi_1(z')-\varphi_2(z')|,\ \sup_{0\le z'\le z}\big|\frac{d}{dz'}\varphi_1(z')-\frac{d}{dz'}\varphi_2(z')\big|\big),$$

for $0\le z\le z_1$,we denote

$$\kappa_{\widetilde{\varrho}}(z)=\kappa(\widetilde{\varrho},\varrho^*_{hs})(z),\qquad \kappa_{\widetilde{T}}(z)=\kappa(\widetilde{T},T^*_{hs})(z),\qquad \kappa_{\widetilde{w}}(z)=\kappa(\widetilde{w},\frac{\widetilde{U}}{\varrho^*_{hs}})(z),$$

$$A_\kappa(z)=\{(\varrho,T,w)/\kappa(\varrho,\widetilde{\varrho})(z')\le\kappa_{\widetilde{\varrho}}(z'),\kappa(T,\widetilde{T})(z')\le\kappa_{\widetilde{T}}(z'),\kappa(w,\widetilde{w})(z')\le\kappa_{\widetilde{w}}(z')\}.$$

We suppose that, if $(\varrho,T,w)\in A_\kappa(z_1)$, we will have

$$\varrho(z)\ge\frac{1}{2}\widetilde{\varrho}(z)\qquad T(z)\ge\frac{1}{2}\widetilde{T}(z)\qquad w(z)\ge\frac{1}{2}\widetilde{w}(z)\qquad\forall z\in[0,z_1].$$

Hypothesis (a) On the interval $[0,z](z\in]0,z_1])$, we suppose that the solution of the system of equations (1.1)–(1.3) with the initial conditions (1.5) belongs to $A_\kappa(z)$.

To show the existence and the uniqueness of the solution of the system of equations (1.1)–(1.3), we pose

$$U(z)=\varrho(z)w(z),\quad V(z)=R_1\varrho(z)T(z)+g\int_0^z\varrho(z')dz',\quad W(z)=\log\frac{T(z)^{c_v}}{\varrho(z)^{R_1}}+\frac{L_{tr}}{\varrho T}\overline{\pi}_{vs}(T).$$

we note that U , V , W verify the equations

$$\frac{d}{dz}(\log U-\log\widetilde{U})=f_1-\widetilde{f}_1,$$

$$\frac{d}{dz}(V-\widetilde{V})=g_1-\widetilde{g}_1,$$

$$\frac{d}{dz}(W-\widetilde{W})=h_1-\widetilde{h}_1,$$

where

$$f_1=-\Big(\overline{\pi}_{vs}(T)\frac{d}{dz}\log\varrho-\frac{d}{dz}\overline{\pi}_{vs}(T)\Big)\frac{1}{\varrho},\qquad g_1=-\alpha^2U\frac{dw}{dz}-g\Sigma$$

$$h_1=\frac{(R_1T+L_{tr})}{\varrho T}\overline{\pi}_{vs}(T)\frac{d}{dz}\log\varrho-\frac{R_1}{\varrho}\frac{d}{dz}\overline{\pi}_{vs}(T)+\frac{d}{dz}\Big(\frac{L_{tr}}{\varrho T}\Big)\overline{\pi}_{vs}(T),\tag{5.1}$$

with the initial conditions

$$\log U(0)-\log\widetilde{U}(0)=0\quad V(0)-\widetilde{V}(0)=0\quad W(0)-\widetilde{W}(0)=0,$$

where $\widetilde{f}_1$, $\widetilde{g}_1$, $\widetilde{h}_1$ are defined in (3.2)–(3.4).
For the functions $\varphi(\varrho,T,w)$ we will also use the notation

$$[\varphi]_\kappa(z)=\sup_{(\varrho,T,w)\in A_\kappa(z),z'\in[0,z]}\Big|\varphi(\varrho,T,w)(z')\Big|$$

Lemma 5.1. *Let* $y(z) = e^{-W(z)/R_1} T(z)^{\frac{R_1+c_v}{R_1}} e^{\left(\frac{L_{tr}\overline{\pi}_{vs}(T)}{R_1 \varrho T}\right)}$, *assuming that (a) is verified we have*

$$|Y(z) - \widetilde{Y}(z)| \leq \int_0^z M_Y(z')dz'. \tag{5.2}$$

$$\left|\frac{d}{dz}Y(z) - \frac{d}{dz}\widetilde{Y}(z)\right| \leq M_Y(z), \tag{5.3}$$

where

$$M_Y(z) = \Big(1 + \frac{\widetilde{g}_1}{\widetilde{y}^{\frac{c_v}{(R_1+c_v)}}} - \frac{g}{R_1} e^{\frac{\widetilde{W}}{(R_1+c_v)}} \Big(\phi(Y_0) + \frac{d}{d\widetilde{Y}}\phi(\widetilde{Y}) \mid_{\widetilde{Y}=Y_0} + R(\widetilde{Y})\Big)\Big)\times$$

$$\times\Big(\frac{2Y_0 g}{R_1} e^{\frac{g}{R_1}\int_0^{z'} \frac{d\phi}{dY}\phi(Y)|_{Y=Y_0} e^{\frac{-W}{(R_1+c_v)}} dz'} \Big(\frac{d\phi}{dY} \mid_{Y=Y_0} \frac{2e^{\frac{-W}{(R_1+c_v)}}}{(R_1+c_v)} \int_0^{z'} (\varphi_h^T \kappa_{\widetilde{T}} + \varphi_h^{\varrho}\kappa_{\widetilde{\varrho}})dz''\Big) +$$

$$e^{\frac{-\widetilde{W}}{R_1+c_v}} \big(\frac{d\phi}{dY} \mid_{Y=Y_0} - \frac{d\phi}{d\widetilde{Y}} \mid_{\widetilde{Y}=Y_0}\big)\Big) + e^{\int_0^{z'} \frac{g}{R_1}\frac{d}{dY}\phi(Y)|_{Y=Y_0} e^{\frac{-W}{(R_1+c_v)}} dz''} \times$$

$$\times\Big(\Big(\frac{c_v|\widetilde{g}_1|}{R_1+c_v}\Big[\frac{1}{\varrho T}\Big]^{\frac{c_v}{R_1+c_v}} \Big(\Big[\frac{1}{\varrho}\Big]_\kappa \kappa_{\widetilde{\varrho}} + \Big[\frac{1}{T}\Big]_\kappa \kappa_{\widetilde{T}}\Big) + \Big[\frac{1}{\varrho T}\Big]_\kappa^{\frac{c_v}{R_1+c_v}} \Big(\phi_g^w \kappa_{\widetilde{w}} + \phi_g^{\varrho}\kappa_{\widetilde{\varrho}}\Big)\Big) +$$

$$+\frac{g}{R_1}\Big(e^{\frac{-W}{(R_1+c_v)}}\Big(Y_0\big(\frac{d}{dY}\phi \mid_{Y=Y_0} - \frac{d}{d\widetilde{Y}}\phi \mid_{\widetilde{Y}=Y_0}\big) + (R(Y) - R(\widetilde{Y}))\Big) +$$

$$+(\phi(Y_0) + \frac{d\phi}{d\widetilde{Y}} \mid_{Y=Y_0} Y_0 + R(\widetilde{Y}))\frac{2e^{\frac{-W}{(R_1+c_v)}}}{(R_1+c_v)} \int_0^{z'} (\varphi_h^T \kappa_{\widetilde{T}} + \varphi_h^{\widetilde{\varrho}}\kappa_{\widetilde{\varrho}})dz''\Big)\Big),$$

and $\phi_g^w, \phi_g^{\varrho}, \varphi_h^T, \varphi_h^{\varrho}, \widetilde{g}_1$ *are defined as follows*

$$|g_1(z) - \widetilde{g}_1(z)| \leq \Phi_g^w \kappa_{\widetilde{w}} + \Phi_g^{\varrho}\kappa_{\widetilde{\varrho}}, \tag{5.4}$$

where

$$\Phi_g^w = 2\alpha^2 \varrho_{hs}^* \Big[\Big|\frac{dw}{dz}\Big|\Big]_\kappa + 2\alpha^2 \widetilde{U},$$

$$\Phi_g^{\varrho} = 2\alpha^2 \Big[\Big|\frac{dw}{dz}\Big|\Big][\widetilde{w}]_\kappa.$$

$$\varphi_h^{\varrho} = \Big[\frac{1}{\varrho}\Big]_\kappa [\overline{\pi}_{vs}(T)]_\kappa \Big(R_1 + L_{tr}\Big[\frac{1}{T}\Big]_\kappa\Big)\Big([\frac{1}{\varrho}]_\kappa + \Big|\frac{d\varrho_{hs}}{dz}\Big|\Big[\frac{1}{\varrho}\Big]_\kappa \frac{1}{\varrho_{hs}^*} + \Big|\frac{d\varrho_{hs}}{dz}\Big|\Big[\frac{1}{\varrho^2}\Big]_\kappa\Big) + \tag{5.5}$$

$$+R_1\Big[\frac{d\overline{\pi}_{vs}}{dT}\Big]_\kappa \Big|\frac{dT_{hs}}{dz}\Big|\Big[\frac{1}{\varrho^2}\Big]_\kappa + L_{tr}\overline{\pi}_{vs}(\widetilde{T})\Big[|\frac{d}{dz}(\frac{1}{\varrho^2 T})|\Big]_\kappa.$$

$$\varphi_h^T = \Big[\overline{\pi}_{vs}(T)\Big]_\kappa \Big(\frac{1}{\varrho_{hs}}|\frac{d\varrho_{hs}}{dz}|\Big(\frac{R_1[T]_\kappa + Ltr}{\widetilde{T}[\varrho T]_\kappa} + \frac{R_1}{\widetilde{\varrho}\widetilde{T}}\Big) + L_{tr}(\frac{d}{dz}|\frac{1}{\widetilde{T}[\varrho T]_\kappa}|)\Big) \tag{5.6}$$

$$+\Big[\frac{d}{dT}\overline{\pi}_{vs}(T)\Big]_\kappa \Big((R_1 + \frac{L_{tr}}{[T]_\kappa})[\frac{1}{\varrho^2}]_\kappa|\frac{d\varrho_{hs}}{dz}| + \frac{R_1}{\widetilde{\varrho}} + L_{tr}[|\frac{d}{dz}(\frac{1}{T\varrho})|]_\kappa\Big) + \frac{1}{\widetilde{\varrho}}[|\frac{dT}{dz}|]_\kappa[\frac{d^2\overline{\pi}_{vs}}{dT^2}]_\kappa.$$

P r o o f. In a similar way to (4.2), we have

$$\frac{d}{dz}Y+\frac{g}{R_1}e^{-\frac{W}{(R_1+c_v)}}\frac{d\phi}{dY}\mid_{Y=Y_0}Y=\frac{1}{R_1}g_1y^{\frac{-c_v}{R_1+c_v}}+\frac{g}{R_1}e^{-\frac{W}{(R_1+c_v)}}\Big(Y_0\frac{d\phi}{dY}\mid_{Y=Y_0}+R(Y)\Big),$$

$$Y(0)=Y_0.$$

The solution of this Cauchy problem is defined by

$$Y(z)=Y_0e^{\int_0^z\frac{g}{R_1}e^{-\frac{W}{(R_1+c_v)}}\frac{d\phi}{dY}|_{Y=Y_0}dz'}+$$

$$+\int_0^z\Big(\frac{1}{R_1}g_1y^{\frac{-c_v}{R_1+c_v}}+\frac{g}{R_1}e^{-\frac{W}{(R_1+c_v)}}\Big(Y_0\frac{d\phi}{dY}\mid_{Y=Y_0}+R(Y)\Big)\Big)e^{\int_0^{z'}\frac{g}{R_1}e^{-\frac{W}{(R_1+c_v)}}\frac{d\phi}{dY}|_{Y=Y_0}dz''}dz'.$$

Hence we obtain

$$Y(z)-\widetilde{Y}(z)=Y_0\Big(e^{\int_0^z\frac{g}{R_1}e^{\frac{-W}{R_1}}\frac{d}{dY}\phi|_{Y=Y_0}}-e^{\int_0^z\frac{g}{R_1}e^{\frac{-\widetilde{W}}{(R_1+c_v)}}\frac{d}{d\widetilde{Y}}\phi|_{\widetilde{Y}=Y_0}}\Big)+ \tag{5.7}$$

$$+\int_0^z\Big[e^{\int_0^z\frac{g}{R_1}e^{\frac{-W}{(R_1+c_v)}}\frac{d}{dY}\phi|_{Y=Y_0}}\Big(\frac{g_1}{R_1y^{\frac{c_v}{R_1+c_v}}}-\frac{g}{R_1}e^{\frac{-W}{(R_1+c_v)}}(\phi(Y_0)+\frac{d}{dY}\phi\mid_{Y=Y_0}Y_0+R(Y))\Big)+$$

$$-e^{\int_0^z\frac{g}{R_1}e^{\frac{-\widetilde{W}}{(R_1+c_v)}}\frac{d}{dY}\phi|_{\widetilde{Y}=Y_0}}\Big(\frac{\widetilde{g}_1}{R_1\widetilde{y}^{\frac{c_v}{R_1+c_v}}}+\frac{g}{R_1}e^{\frac{-\widetilde{W}}{(R_1+c_v)}}(\phi(Y_0)+\frac{d}{d\widetilde{Y}}\phi\mid_{\widetilde{Y}=Y_0}Y_0+R(\widetilde{Y}))\Big)\Big]dz'.$$

From (3.3) we obtain (5.4).

On the other hand, taking into account the relations $h_{tr}\geq 0$, and (3.4) we obtain

$$|e^{-\frac{H_1(z')}{R_1+c_v}}-e^{-\frac{\widetilde{H}_1(z')}{R_1+c_v}}|\leq\frac{2\exp\Big(\frac{-H_1}{R_1+c_v}\Big)}{R_1+c_v}\int_0^z(\varphi_h^{\varrho}\kappa_{\widetilde{\varrho}}+\varphi_h^{T}\kappa_{\widetilde{T}})dz', \tag{5.8}$$

where

$$H_1(z)=\int_0^z h_1(z')dz',\qquad \widetilde{H}_1(z)=\int_0^z\widetilde{h}_1(z')dz',$$

and φ_h^{ϱ}, φ_h^{T} are defined by (5.5), (5.6).

By using $y(z)=\varrho(z)T(z)$, $\widetilde{y}(z)=\widetilde{\varrho}(z)\widetilde{T}(z)$, from (5.7), (5.4), (5.8) we obtain (5.2) and (5.3). □

Lemma 5.2. *We suppose that hypothesis (a) is satisfied, then we have*

$$|T-\widetilde{T}|\leq\Psi_T^0(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})(z), \tag{5.9}$$

$$|\frac{dT}{dz}-\frac{d\widetilde{T}}{dz}|\leq\Psi_T^1(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})(z), \tag{5.10}$$

$$|\varrho-\widetilde{\varrho}|\leq\Psi_\varrho^0(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})(z), \tag{5.11}$$

$$|\frac{d\varrho}{dz}-\frac{d\widetilde{\varrho}}{dz}|\leq\Psi_\varrho^1(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})(z). \tag{5.12}$$

$$|w(z)-\widetilde{w}(z)|\leq\Psi_w^0(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})(z). \tag{5.13}$$

$$|\frac{dw}{dz}(z)-\frac{d\widetilde{w}}{dz}(z)|\leq\Psi^1_w(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})(z). \tag{5.14}$$

Where

$$\Psi^0_T(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})=\frac{1}{\frac{d}{dT}(Z(T^*))}\int_0^z M_y(z')dz'.$$

$$\Psi^1_T(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})=\frac{1}{(\frac{d}{dT}Z(T^*))^2}\Big(\frac{d}{dT}Z(T^*)M_Y(z)-\frac{d^2}{dT^2}Z(T^*)\int_0^z M_Y(z')dz'\Big)$$

$$\Psi^0_\varrho(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})=[\varrho T]_\kappa\frac{1}{\widetilde{T}[T]_\kappa}\Psi^0_T+\frac{R_1+c_v}{R_1}\frac{1}{\widetilde{\widetilde{T}}}[\varrho T]_\kappa^{\frac{c_v}{R_1+c_v}}\int_0^z M_y(z')dz'.$$

$$\Psi^1_\varrho(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})=\big[|\frac{d}{dz}(\frac{1}{[T]\widetilde{T}})|\big]_\kappa\Psi^0_T+\big[\frac{1}{[T]_\kappa\widetilde{T}}\big]_\kappa\Psi^1_T+\frac{R_1+c_v}{R_1}\int_0^z M_Y(z')dz'\big(\frac{d}{dz}(\frac{1}{\widetilde{\widetilde{T}}})[\varrho T]_\kappa^{\frac{c_v}{R_1+c_v}}+$$

$$+\frac{1}{\widetilde{\widetilde{T}}}\frac{d}{dz}[\varrho T]_\kappa^{\frac{c_v}{c_v+R_1}}\big)+\frac{R_1+c_v}{R_1}\frac{1}{\widetilde{\widetilde{T}}}[\varrho T]_\kappa M_Y(z).$$

$$\Psi^0_w(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})=\frac{e^{[F_1]_\kappa}}{\widetilde{\varrho}}\Big[\frac{1}{\varrho}\Big]_\kappa\Psi^0_\varrho(z)+2e^{[F_1]_\kappa}\Big[\frac{1}{\varrho}\Big]\times\int_0^z\big(\varphi^T_f\kappa_T+\varphi^\varrho_f\kappa_\varrho\big)dz'.$$

$$\Psi^1_w(\kappa_{\widetilde{\varrho}},\kappa_{\widetilde{T}},\kappa_{\widetilde{w}})=\big(\frac{1}{\widetilde{\varrho}[\varrho]_\kappa}[f_1]_\kappa+\frac{1}{\widetilde{\varrho}[\varrho^2]_\kappa}[|\frac{d\varrho}{dz}|]_\kappa+\frac{1}{\widetilde{\varrho}^2[\varrho]_\kappa}[|\frac{d\widetilde{\varrho}}{dz}|]_\kappa\big)e^{[F_1]_\kappa}\Psi^0_\varrho+\frac{e^{[F_1]_\kappa}}{\widetilde{\varrho}[\varrho]_\kappa}\Psi^1_\varrho+$$

$$+2e^{[F_1]_\kappa}(\frac{d}{dz}\big(\frac{1}{[\varrho]_\kappa}\big)+\frac{[f]_\kappa}{[\varrho]_\kappa})\int_0^z(\varphi^T_f\kappa_T+\varphi^\varrho_f\kappa_\varrho)dz'+\big[\frac{1}{\varrho}\big]_\kappa e^{\widetilde{F}_1}(\varphi^T_f\kappa_T+\varphi^\varrho_f\kappa_\varrho),$$

such as $Z(T)$ *,* $\varphi^T_{f_1}$ *,* $\varphi^\varrho_{f_1}$ *are defined as follows*

$$Z(T)=Te^{\frac{L_{tr}\overline{\pi}_{vs}(T)}{(R_1+c_v)\varrho T}},$$

$$\varphi^T_f=\Big((2\Big[\frac{1}{\varrho^3}\Big]_\kappa\Big[|\frac{d\varrho}{dz}|\Big]_\kappa+\Big[\frac{1}{\varrho^2}\Big]_\kappa)+\Big[\frac{1}{\varrho^2}\Big]_\kappa\Big[|\frac{dT}{dz}|\Big]_\kappa\Big[\frac{d}{dT}\overline{\pi}_{vs}(T)\Big]_\kappa\Big), \tag{5.15}$$

$$\varphi^\varrho_f=\Big((\Big[\frac{1}{\varrho^2}\Big]_\kappa\Big[|\frac{d\varrho}{dz}|\Big]_\kappa+\Big[\frac{1}{\varrho}\Big]_\kappa)\Big[\frac{d}{dT}\overline{\pi}_{vs}(T)\Big]_\kappa+\Big[\frac{1}{\varrho}\Big]_\kappa+\Big[|\frac{dT}{dz}|\Big]_\kappa\Big[\frac{d^2}{dT^2}\overline{\pi}_{vs}(T)\Big]_\kappa\Big). \tag{5.16}$$

P r o o f. We define the function

$$Z(T)=Te^{\frac{L_{tr}\overline{\pi}_{vs}(T)}{(R_1+c_v)\varrho T}},$$

so we have

$$T=Z^{-1}(Te^{\frac{L_{tr}\overline{\pi}_{vs}(T)}{(R_1+c_v)\varrho T}}), \tag{5.17}$$

Taking into account the relation (5.17),we have

$$T(z)-\widetilde{T}(z)=\frac{d}{dz}Z^{-1}(Z^*)(Y-\widetilde{Y})=\frac{Y-\widetilde{Y}}{\frac{d}{dT}Z(T^*)},$$

with

$$\frac{d}{dT}Z(T)=e^{\frac{L_{tr}\overline{\pi}_{vs}}{(R_1+c_v)\varrho T}}\Big(1+\frac{L_{tr}\frac{d\overline{\pi}_{vs}}{dT}}{(R_1+c_v)\varrho}-\frac{L_{tr}\overline{\pi}_{vs}(T)\frac{dT}{dz}}{(R_1+c_v)\varrho T}\Big).$$

Therefore, according to (5.2) we obtain (5.9).

In addition, we have

$$\frac{dT}{dz}-\frac{d\widetilde{T}}{dz}=\frac{1}{(\frac{d}{dT}Z(T^*))^2}\Big(\frac{d}{dT}Z(T^*)M_Y(z)-\frac{d^2}{dT^2}Z(T^*)\int_0^z M_Y(z')dz'\Big).$$

Similarly to estimate (5.9), we obtain (5.10).

For estimation (5.11) we have

$$\varrho=e^{-w/R_1}(Te^{\frac{L_{tr}\overline{\pi}vs}{(R_1+c_v)\varrho T}})^{\frac{c_v}{R_1}}e^{\frac{L_{tr}\overline{\pi}vs}{(R_1+c_v)\varrho T}}.$$

With a simple calculation we obtain

$$\varrho=e^{-w/R_1}T^{\frac{R_1+c_v}{R_1}}e^{\frac{L_{tr}\overline{\pi}vs}{(R_1+c_v)\varrho T}}\frac{1}{T}=y\frac{1}{T},$$

so

$$\varrho-\widetilde{\varrho}=y(\frac{1}{T}-\frac{1}{\widetilde{T}})+\frac{1}{\widetilde{T}}(y-\widetilde{y}).$$

In addition we have

$$\frac{d\varrho}{dz}-\frac{d\widetilde{\varrho}}{dz}=\frac{d}{dz}(\frac{1}{\widetilde{T}T})(T-\widetilde{T})+\frac{1}{T\widetilde{T}}\frac{d}{dz}(T-\widetilde{T})+$$
$$\frac{R_1+c_v}{R_1}\Big(\frac{d}{dz}(\frac{1}{\widetilde{T}})[y]^{\frac{c_v}{R_1+c_v}}(Y-\widetilde{Y})+\frac{1}{\widetilde{T}}\big(\frac{d}{dz}y^{\frac{c_v}{(R_1+c_v)}}(Y-\widetilde{Y})+[y]_\kappa^{\frac{c_v}{R_1+c_v}}\frac{d}{dz}(Y-\widetilde{Y})\big)\Big),$$

and

$$(Y-\widetilde{Y})=\frac{R_1}{R_1+c_v}y^{\frac{-c_v}{R_1+c_v}}(y-\widetilde{y}). \tag{5.18}$$

From (5.18), (5.9), (5.10) we obtain (5.11), (5.12).

To obtain (5.13) and (5.14), we denote by:

$$F_1(z)=\int_0^z f_1(z')dz' \qquad \widetilde{F}_1(z)=\int_0^z \widetilde{f}_1(z')dz'.$$

According to (3.2)–(4.1), we have

$$w(z)-\widetilde{w}(z)=\big(\frac{e^{F_1}}{\varrho\widetilde{\varrho}}(\widetilde{\varrho}-\varrho)+\frac{1}{\varrho}(e^{F_1}-e^{\widetilde{F}_1})\big).$$

$$\frac{dw}{dz}-\frac{d\widetilde{w}}{dz}=\frac{d}{dz}(\frac{e^{F_1}}{\varrho\widetilde{\varrho}})(\widetilde{\varrho}-\varrho)+(\frac{e^{F_1}}{\varrho\widetilde{\varrho}})\frac{d}{dz}(\widetilde{\varrho}-\varrho)+$$
$$+\big[e^{F_1}(\widetilde{\varrho}-\varrho)(\frac{f_1}{\varrho\widetilde{\varrho}}+\frac{d}{dz}\frac{1}{\varrho\widetilde{\varrho}})+(e^{F_1}-e^{\widetilde{F}_1})\frac{d}{dz}\frac{1}{\varrho}+\frac{1}{\varrho}(e^{F_1}f_1-e^{\widetilde{F}_1}\widetilde{f}_1)\big]$$

From (1.4), (3.1), (5.1), we have

$$F_1-\widetilde{F}_1=2e^{[F_1]_\kappa}\int_0^z(\varphi_f^T\kappa_T+\varphi_f^\varrho\kappa_\varrho)dz',$$

where φ_f^T, φ_f^ϱ are defined in (5.15), (5.16).

Thus we find (5.13), (5.14). □

Theorem 5.1. *Let* $0 < z_1 < \overline{z_1}$. *If there is* $\overline{\varepsilon}_0$ *such as* $0 < \overline{\varepsilon}_0 < 1$ *and the relations*

$$\max(\Psi^0_\varrho(\kappa_{\widetilde{\varrho}}, \kappa_{\widetilde{T}}, \kappa_{\widetilde{w}})(z), \Psi^1_\varrho(\kappa_{\widetilde{\varrho}}, \kappa_{\widetilde{T}}, \kappa_{\widetilde{w}})(z)) \le (1-\overline{\varepsilon}_0)\kappa_{\widetilde{\varrho}}(z) \tag{5.19}$$

$$\max(\Psi^0_T(\kappa_{\widetilde{\varrho}}, \kappa_{\widetilde{T}}, \kappa_{\widetilde{w}})(z), \Psi^1_T(\kappa_{\widetilde{\varrho}}, \kappa_{\widetilde{T}}, \kappa_{\widetilde{w}})(z)) \le (1-\overline{\varepsilon}_0)\kappa_{\widetilde{T}}(z), \tag{5.20}$$

$$\max(\Psi^0_w(\kappa_{\widetilde{\varrho}}, \kappa_{\widetilde{T}}, \kappa_{\widetilde{w}})(z), \Psi^1_w(\kappa_{\widetilde{\varrho}}, \kappa_{\widetilde{T}}, \kappa_{\widetilde{w}})(z)) \le (1-\overline{\varepsilon}_0)\kappa_{\widetilde{w}}(z) \tag{5.21}$$

are verified for all $z \in [0, z_1]$, *then there exists a unique solution for the system* (1.1)–(1.3) *with initial conditions* (1.5) *in* $[0, z_1]$.

P r o o f. We consider the system for (ϱ, T, w) and $(\widetilde{\varrho}, \widetilde{T}, \widetilde{w})$ which is expressed by the system $(\varrho^*_{hs}, T^*_{hs}, \frac{\widetilde{U}}{\varrho^*_{hs}})$ for $z = 0$. We have

$$\varrho(0) = \widetilde{\varrho}(0) = \varrho^*_{hs}(0) = \varrho_0,\ T(0) = \widetilde{T}(0) = T^*_{hs}(0) = T_0, w(0) = \widetilde{w}(0) = w^*_{hs}(0) = w_0, \tag{5.22}$$

(where $\varrho^*_{hs}(0) = \varrho^*_{hs}(z(0)), T^*_{hs}(0) = T^*_{hs}(z(0))$), using the expressions h_{tr} , g_1 , $\widetilde{g}_1$ in $z = 0$, we have

$$w_0\frac{d\varrho}{dz} + \varrho_0\frac{dw}{dz} = w_0\frac{d\widetilde{\varrho}}{dz} + \varrho_0\frac{d\widetilde{w}}{dz} = w_0\frac{d\varrho^*_{hs}}{dz} + \varrho_0\frac{d}{dz}\frac{\widetilde{U}}{\varrho^*_{hs}} = -w_0 h_{tr}(0)$$

$$-R_1T_0\frac{d\varrho}{dz} + c_v\varrho_0\frac{dT}{dz} = -R_1T_0\frac{d\widetilde{\varrho}}{dz} + c_v\varrho_0\frac{d\widetilde{T}}{dz} = -R_1T_0\frac{d\varrho^*_{hs}}{dz} + c_v\varrho_0\frac{dT^*_{hs}}{dz} = (R_1T_0 + L_{tr})h_{tr}(0)$$

$$R_1T_0\frac{d\varrho}{dz} + R_1\varrho_0\frac{dT}{dz} = R_1T_0\frac{d\widetilde{\varrho}}{dz} + R_1\varrho_0\frac{d\widetilde{T}}{dz} = -g\varrho_0 + g_1(0)(= -g\varrho_0 + \widetilde{g}_1(0)),$$

$$R_1T_0\frac{d\varrho_{hs}}{dz} + R_1\varrho_0\frac{dT_{hs}}{dz} = -g\varrho_0.$$

After a simple calculation we obtain

$$\frac{d\varrho}{dz}\Big|_{z=0} = \frac{d\widetilde{\varrho}}{dz}\Big|_{z=0}, \qquad \frac{dT}{dz}\Big|_{z=0} = \frac{d\widetilde{T}}{dz}\Big|_{z=0}, \qquad \frac{dw}{dz}\Big|_{z=0} = \frac{d\widetilde{w}}{dz}\Big|_{z=0}. \tag{5.23}$$

On the other hand, we have

$$w_0\frac{d}{dz}(\widetilde{\varrho} - \varrho^*_{hs})\Big|_{z=0} + \varrho_0\frac{d}{dz}(\widetilde{w} - \frac{\widetilde{U}}{\varrho^*_{hs}}) = 0,$$

$$-R_1T_0\frac{d}{dz}(\widetilde{\varrho} - \varrho^*_{hs})\Big|_{z=0} + c_v\varrho_0\frac{d}{dz}(\widetilde{T} - T^*_{hs})\Big|_{z=0} = 0,$$

$$R_1T_0\frac{d}{dz}(\widetilde{\varrho} - \varrho^*_{hs})\Big|_{z=0} + R_1\varrho_0\frac{d}{dz}(\widetilde{T} - T^*_{hs})\Big|_{z=0} = \widetilde{g}_1(0),$$

by solving this system we obtain,

$$\frac{d\widetilde{\varrho}}{dz}\Big|_{z=0} - \frac{d\varrho^*_{hs}}{dz}\Big|_{z=0} = \frac{c_v\widetilde{g}_1(0)}{R_1T_0(R_1 + c_v)}, \quad \frac{d\widetilde{T}}{dz}\Big|_{z=0} - \frac{dT^*_{hs}}{dz}\Big|_{z=0} = \frac{\widetilde{g}_1(0)}{\varrho_0(R_1 + c_v)},$$

$$\frac{d\widetilde{w}}{dz}\Big|_{z=0} - \frac{d}{dz}(\frac{\widetilde{U}}{\varrho^*_{hs}})\Big|_{z=0} = \frac{c_v w_0\widetilde{g}_1(0)}{R_1\varrho_0T_0(R_1 + c_v)}.$$

It is clear that $\tilde{g}_1 \neq 0$, so from (5.22) and (5.23) it follows that there exists $z_\varepsilon > 0$ so that the hypothesis (a) is verified on the interval $[0, z_\varepsilon]$.

Therefore, according to Lemma 5.2 and relations (5.19)–(5.21) the solution (w, ϱ, T) can be extended until z_1. Which proves Theorem 1.

The uniqueness of the solution results from the uniqueness of the local solution. □

Corollary 5.1. *Let* $0 = z_0 < z_1 < \cdots < z_{n-1} < z_n = \overline{z}_1$. *We suppose that*
i) the relations (5.19)–(5.21) *with initial conditions* $\varrho_0 = \varrho(z_0), T_0 = T(z_0), w_0 = w(z_0)$ *are verified for all* $z \in [z_0, z_1]$,
ii) if $\varrho(z_i)$, $T(z_i)$, $w(z_i)$ *are the values in* $z = z_i$ *of solution of system* (1.1)–(1.3) *in the interval* $[z_{i-1}, z_i]$, *relations* (5.19)–(5.21) *with initial conditions* $\varrho_0 = \varrho(z_i), T_0 = T(z_i)$, $w_0 = w(z_i)$ *are verified for all* $z \in [z_i, z_{i+1}]$, $i = 1, \cdots, n-1$.
Then there is a unique solution of system (1.1)–(1.3) *with initial conditions* (1.5) *in* $[0, \overline{z}_1]$.

P r o o f. By applying Theorem 5.1 successively on each subinterval $[z_i, z_{i+1}]$, $i = 0, \cdots, n-1$, there exists a unique solution of system (1.1)–(1.3) with initial conditions (1.5) in $[0, \overline{z}_1]$. □

This corollary is a trivial consequence of Theorem 1, but in practice we are usually obliged to use it, because the interval of the existence of the solution that the theorem guarantees is often too small, while the actual solution exists in relatively longer interval.

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ИССЛЕДОВАНИЕ СУЩЕСТВОВАНИЯ ГЛОБАЛЬНОГО РЕШЕНИЯ СИСТЕМЫ УРАВНЕНИЙ, ОПИСЫВАЮЩЕЙ ВЕРТИКАЛЬНОЕ ДВИЖЕНИЕ ВОЗДУХА В ДЫМОХОДЕ

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Аннотация. Рассматривается система уравнений, описывающая движение воздуха в дымоходе. Предлагается метод, основанный на «втором приближении». Доказана теорема существования и единственности «глобального решения».
Ключевые слова: глобальное решение; движение воздуха

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